Have-you-seen...

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respective numbers.... Each State shall have at least one representative."

Suppose a state has 10 percent of the population. Since there are 435 seats in the HR, the state should be entitled to 43.5 seats. The problem is how to treat the fractions which inevitably arise! Mathematics has been used since the beginnings of our nation to analyze how the seats in the HR should be apportioned. The history of this interaction has been long and fascinating; see "An Apportionment Problem" at the right of this page for a description of the methods of Alexander Hamilton and Thomas Jefferson and see the mini-bibliography on page 9 for further references.

The method currently in use is known as the Hill-Huntington Method (or, in a misnomer, the method of Equal Proportions). However, on December 17, 1991 the New York Times carried an article about the decision of a United States District Court, involving the state of Montana, declaring the method unconstitutional! (See [1] and also [2], [3], [4].) The Supreme Court immediately decided to hear the case, since state redistricting decisions and the legislative process itself depend on knowing that the HR is legally constituted.

In a related development, a Federal Court allowed the state of Massachusetts to maintain 11 seats in the HR rather than have one of its seats given to Washington. The decision was based on the fact that giving the seat to Washington was based on census data which included Americans living abroad. It was successfully argued by Massachusetts that data for Americans living abroad was so unreliable that only domestic data should be used. Based on domestic data only, Massachusetts was entitled to maintain 11 seats. (See [5], [6])

The Supreme Court, by votes of 9-0 in each case, overturned both the lower court decisions [8,9,10,11]. In an April 1st article in the NY Times dealing with the Supreme Court decision in the Montana case [8], it is stated that in his decision Justice Stevens was of the opinion that the method of equal proportions led to the least bias with regard to the number of seats given to small and large states. In fact, the relatively recent mathematical research of Balinski and Young [1, mini-bibliography] shows that Justice Stevens is mistaken.

Understanding the mathematics of apportionment begins with arithmetic and opens up exciting ideas in the field of discrete optimization. ■

References:

1. **High Court to Weigh Redistricting Case.** NY Times, Dec. 17, 1991, pg. A18.
2. **Court to hear Montana suit on House.** Ruth Marcus. The Washington Post, Dec. 17, 1991, v115 pg. A4
3. **High court to weigh constitutionality of apportioning House seats to states.** Paul M. Barrett. The Wall Street Journal, Dec. 17,1991, pgs. A16(W) & A22(E)
4. **Remapping of the States Up to the Courts.** LA Times, September 13, 1991.
5. **US Panel allows Massachusetts to Retain its 11 Representatives.** NY Times, February 21, 1992, pg. A16.
6. **Justices to hear Massachusetts case involving census.** Paul M. Barrett. The Wall Street Journal, March 21, 1992, pgs. B3C(W) & B6B(E)
7. **Supreme Court agrees to hear census dispute; number of House seats for 2 states is at stake.** Linda Greenhouse. The New York Times, March 21, 1992, v141 pg. 6
8. **Supreme Court upholds method used in apportionment of House.** (National Pages) Linda Greenhouse. The New York Times, April 1, 1992, v141 pgs. A18(N) & B8(L)
9. **Supreme Court decision ends Montana bid to keep House seat.** David G. Savage. Los Angeles Times, April 1, 1992, v111 pg. A10
10. **Justices back method of apportionment used to distribute House seats to states.** Paul M. Barrett. The Wall Street Journal, April 1, 1992, pg. A14(W), pg, A18(E)
11. **Massachusetts loses House seat as Washington State gains.** The New York Times, June 27, 1992, v 141, pg. 11

An Apportionment Problem

Suppose there are 10 seats in the legislature for a country having 3 states (named A, B, and C) having populations 740, 170, and 90, respectively. How many seats should each state get in the HR if each state must get at least one seat?

Alexander Hamilton's method (the method of largest remainders) would work in this way. Since A, B, and C have 74, 17, and 9 percent of the population, begin by computing .74(10) = 7.4, .17(10) = 1.7, and .09(10) = .9 (i.e. the percentage population times the size of the legislature). We assign each state the integer part of this product. Thus, A gets 7 seats, B gets 1 seat and C gets 0 seats. Since 7 + 1 + 0 = 8, we must assign 2 more seats. Order the fractional remainders above in decreasing order: .9(C), .7(B), and .4(A). The two extra seats go to C and B in this order, so that the final apportionment is A = 7, B = 2, and C = 1. B and C are over-represented and A is under-represented.

Here is a different approach using the same data. It is sometimes called Jefferson's method. Consider the table on page 9. The first line consists of the original population P of each state. The second line consists of the original data divided by 2, the third line consists of the original data divided by 3, and so on.

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